AN2874
Applications note

BQD filter design equations

1 Introduction

A generalized set of equations is formulated for the design of first-order and second-order low-pass and high-pass filters. A specialized set of equations is then given for the design of parametric biquad EQ filters.

The parameters governing the characteristics of the filter are:

- filter cut-off frequency, f_C (-3 dB corner frequency)
- sampling rate (f_S).
- Q factor
- boost/cut gain value at f_C.

These parameters can be used to determine the coefficients of the filter transfer function.

The transfer function for a first-order filter in the digital z-domain can be written as:

$$H(z) = (b_0 + b_1.z^{-1}) / (a_0 + a_1.z^{-1}).$$

And for a second-order filter as:

$$H(z) = (b_0 + b_1.z^{-1} + b_2.z^{-2}) / (a_0 + a_1.z^{-1} + a_2.z^{-2}).$$

This document can be used in conjunction with all the ST digital audio devices having EAQ filters onboard. These include STA309A, STA320, STA321, STA33xBW and several others.

2 Low-pass and high-pass filter design

First-order filter design

As a first step to obtain the coefficients for the 1st-order low-pass or high-pass filter equation on page 1, the following equations can be used:

$\theta_C = 2 * \pi * f_C / f_S$ (= the normalized cut-off frequency)

$K = \tan(\theta_C / 2)$

$\alpha = 1 + K$.

The denominator coefficients are identical for both low-pass and high-pass filters designed for the same cut-off frequency and are computed as follows:

$a_0 = 1$

$a_1 = -(1 - K) / \alpha$.

The numerator coefficients for a low-pass filter can be calculated as follows:

$b_0 = K / \alpha$

$b_1 = K / \alpha$.

The numerator coefficients for a high-pass filter can be calculated as follows:

$b_0 = 1 / \alpha$

$b_1 = -1 / \alpha$.

Second-order filter design

As a first step to obtain the coefficients for the 2nd-order low-pass or high-pass filter equation on page 1, the following equations can be used:

$\theta_C = 2 * \pi * f_C / f_S$

$K = \tan(\theta_C / 2)$

$W = K^2$

$\alpha = 1 + K / Q + W$.

The denominator coefficients are the same for both low-pass and high-pass filters designed for the same cut-off frequency and are computed as follows:

$a_0 = 1$

$a_1 = 2 * (W - 1) / \alpha$

$a_2 = (1 - K / Q + W) / \alpha$.

The numerator coefficients for a low-pass filter can be calculated as follows:

$b_0 = W / \alpha$

$b_1 = 2 * W / \alpha$

$b_2 = b_0$.

The numerator coefficients for a high-pass filter can be calculated as follow:

$b_0 = 1 / \alpha$

$b_1 = -2 / \alpha$

$b_2 = b_0$.

3   Parametric EQ filter design

The transfer function of a 2nd-order parametric EQ filter is identical to the one used for 2nd-order low-pass or high-pass filter design. Parametric EQ filters are used for the emphasis of frequency components around the desired edge or corner frequency. An additional piece of data is needed that specifies the amount of boost/cut required around the corner frequency:

$$\theta_C = 2 * \pi * f_C / f_S$$
$$gF = 10^{G/20} \text{ where } G = \text{boost/cut in dB at } f_C$$
$$\beta = (2 * \theta_C / Q) + \theta_C^2 + 4.$$

The filter coefficients for a 2nd-order parametric EQ filter can be easily calculated by using the above mentioned data in the following set of equations:

$$b0 = (2 * gF * \theta_C / Q + \theta_C^2 + 4) / \beta$$
$$b1 = [(2 * \theta_C^2 - 8) / \beta$$
$$b2 = (4 - 2 * gF * \theta_C / Q + \theta_C^2) / \beta$$
$$a0 = \beta/\beta = 1$$
$$a1 = (2 * \theta_C^2 - 8) / \beta$$
$$a2 = (4 - 2 * \theta_C / Q + \theta_C^2) / \beta.$$

Filter stability conditions

For a 2nd-order filter, two conditions need to be satisfied to ensure filter stability. A filter is said to be stable in the z-domain if the roots (or poles) of the filter lie inside the unit circle. This definition of stability can be translated in terms of the filter coefficients to take the form:

$$|a_2| < 1$$
$$|a_1| < 1 + a_2 \text{ (that is, a1} < 1 + a_2 \text{ AND } a_1 > -(1 + a_2)).$$

For a 1st-order filter, the stability condition that needs to be satisfied is that the pole of the filter lies within the unit circle. Again writing in terms of the coefficients just designed, the condition can be given as:

$$|a_1| < 1.$$

The first-order and second-order systems are stable if and only if the requisite conditions have been satisfied.

4 Filter coefficient formatting for STA309X

The filter coefficients obtained using the design equations are not compatible with the format used by the STA309X GUI applications. The coefficients are in the form of signed fractional numbers, whereas the GUI requires the coefficients to be in a signed fixed-point integer format.

STA309X biquad coefficients

The STA309X uses the following equation for biquad filter processing:

$$y[n] = x[n] + (b_0 - 1).x[n] + 2 * (b_1 / 2).x[n-1] + b_2.x[n-2] - 2 * (a_1 / 2).y[n-1] - a_2.y[n-2]$$
$$= b_0.x[n] + b_1.x[n-1] + b_2.x[n-2] - a_1.y[n-1] - a_2.y[n-2].$$

The order and format in which the coefficients are to be provided is as follows:

b_2

$b_0 - 1$

a_2

$a_1 / 2$

$b_1 / 2$.

The coefficients are constrained to be in a 24-bit signed fixed-point integer format. Hence the filter coefficients must be in the range [0x800000, 0x7FFFFF], which is equivalent to the fractional range [-1.0, 1.0]. In order to meet the afore mentioned criteria, the filter coefficients computed using the design equations are first processed as per the relations given above. This set of newly processed coefficients are quantized to 20-bit fixed-point numbers and then converted into hexadecimal format. Now, the coefficients are ready to be loaded into the STA309X design using its GUI.

5 Revision history

Table 1. Document revision history

Date	Revision	Changes
16-Feb-2009	1	Initial release.

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